B. L. R. Shawyer and B. B. Watson, *Borel's Methods of Summability*, Oxford Mathematical Monographs, Oxford Science Publications, 1994, xii + 242 pp.

This book deals with a family of summability methods closely related to the famous method of Borel. We say that a complex sequence (s_n) converges to s in the sense of Borel, in short $s_n \to s\ (B)$, if $p_s(x) = \sum s_k(x^k/k!)$ converges for all $x \in \mathbb{R}$, and $\sigma(x) := e^{-x} p_s(x) \to s$ as $x \to \infty$. This is a more general notion of convergence. In particular if (s_n) converges to s in the ordinary sense then $s_n \to s\ (B)$. This method is well understood and is relevant to many problems in mathematics.

This book deals with the following topics. After a nice historical overview (Section 1), the authors introduce in Section 2 the notion of summability methods and illuminate it by discussing three families of methods: Cesàro, power series, and Euler methods. In Sections 3 and 4 a collection of well-known classical results about Borel summability, and relations to other methods such as Cesàro and Euler methods, and then in particular the circle methods are presented. To this point the results are presented mostly without proofs. Starting with Section 5 the authors consider a natural generalization of Borel's method, the so-called Borel-type summability methods $B(\alpha, \beta)$, where the transformation of (s_n) is given by

$$\sigma(x) = \alpha e^{-x} \sum_{n=N}^{\infty} s_n x^{\alpha n + \beta - 1} / \Gamma(\alpha n + \beta) \qquad \alpha > 0, \beta \in \mathbb{R},\ N \in \mathbb{N},\ \alpha N + \beta > 0.$$

From this point on proofs are generally included. After a discussion of these methods, the book deals with the interrelationship between $B(\alpha, \beta)$-methods for different α, β (Section 5), Abelian theorems (Section 6), Tauberian theorems—local Tauberian conditions (Section 7), oscillation conditions and the deep gap-Tauberian theorem (Section 8). However, with respect to the latter no proof is provided. Finally, in Section 9 the relationship with other methods is investigated. In the last section some applications to the field of entire functions and arithmetical functions are presented.

The book is a collection of many interesting results related to Borel-type methods which are scattered throughout the literature. It includes an extensive bibliography which is a good source for relevant papers. The text is well written and I enjoyed reading it. There are only a few points which could be criticized. Some of the more recent results in the field are not included, e.g., the Tauberian theorems are true for a much more general class of summability methods, without being more complicated (results of D. Borwein, W. Kratz, and the reviewer). Some interesting relations to other fields in mathematics are not mentioned, e.g., for the asymptotics some probability is useful and natural (local central limit theorems). The applications range from probability to physics and some discussion of these would have made the book more interesting for the nonspecialist.

Ulrich Stadtmüller

G. E. Andrews, B. C. Berndt, L. Jacobsen, and R. L. Lamphere, *The Continued Fractions Found in the Unorganized Portions of Ramanujan's Notebooks*, Memoirs of the American Mathematical Society **477**, Amer. Math. Soc., Providence, RI, 1992, vi + 71 pp.

The authors discuss 60 continued fraction entries in 133 pages of the unorganized portions of Ramanujan's second and third notebooks (published in two volumes by the Tata Institute of Fundamental Research, Bombay, 1957). Many of these entries are related to previous entries in the organized portions of the second notebook, especially Chapters 12 and 16 which were examined by Berndt, Lamphere, and Wilson [*Chapter* 12 *of Ramanujan's second notebook: continued fractions*, *Rocky Mountain J. Math.* **15** (1985), 235–310] and by Adiga, Berndt, Bhargava, and Watson [*Chapter* 16 *of Ramanujan's notebook: theta-functions and q-series*, *Mem. Amer. Math. Soc.* **315**, 1985]. Approximately one-third of the entries are q-continued fractions. Several of these are related to Ramanujan's only published continued fraction, the famous Rogers–Ramanujan continued fraction. However, most of the entries are new. For each entry the authors give a proof and/or provide a reference to a proof. They also discuss connections with other entries and with the works of others.

This is a scholarly work that requires careful reading and checking in order to be fully appreciated. However, there is nothing too technical to prevent it from being read by someone with little or no knowledge of continued fractions. Both the expert and non-expert will profit from studying its contents and will be sure to become Ramanujan fans. It is recommended reading for anyone interested in special functions or approximations and expansions.

DAVID R. MASSON

Y. XU, *Common Zeros of Polynomials in Several Variables and Higher Dimensional Quadrature*, Pitman Research Notes in Mathematics Series **312**, Longman Scientific & Technical, Essex (U.K.), 1994.

This book collects some significant recent results in the theory of multidimensional quadrature formulas. Given a square positive functional $\mathscr{L}(f)$ on the space of the multivariate polynomials, a quadrature formula is an approximation of this functional of the form $\sum_{k=1}^{N} \lambda_k f(\mathbf{x}_k)$ that is exact for all polynomials f up to a certain degree. The nodes $\mathbf{x}_k$ are restricted to be real and the weights λ_k to be positive.

As in the one-dimensional case, the theory of quadrature formulas is based on the theory of orthogonal polynomials. The author uses a compact vector notation for the orthogonal polynomials and in a preliminary chapter he reviews their theory. The nodes of the quadrature formula are described as the common zeros of a set of quasi-orthogonal multivariate polynomials, which depend on matrices of parameters. Rather than relying on algebraic geometry for studying the common zeros of these polynomials, the author constructs properly tailored truncated block Jacobi matrices so that the common zeros of the polynomials are joint eigenvalues of these matrices. This leads to a characterization of the sets of quasi-orthogonal polynomials that generate interpolatory quadrature formulas with real nodes and positive weights. This characterization takes the form of nonlinear matrix equations in the parameters of these sets. Theorems 4.1.4 and 7.1.4 are, however, not properly formulated so that the reader may wrongly think that it is a characterization of the sets of polynomials whose common zeros are all real. The approach of the author is general; it deals with quadrature formulas of odd degree as well as of even degree. The relation with Möller's lower bound is explored in a separate chapter. Another chapter illustrates the theory with examples for which the nonlinear matrix equations can be solved.

The book is a research paper recommended to persons interested in the theoretical aspects of multidimensional quadrature and in multivariate orthogonal polynomials. It is self-contained and assumes no specific knowledge.

PIERRE VERLINDEN

K. KITAHARA, *Spaces of Approximating Functions with Haar-like Conditions*, Lecture Notes in Mathematics **1576**, Springer-Verlag, 1994, x + 110 pp.

The book contains five chapters and two appendices, with each chapter containing a problems section. A considerable portion of the book is based on the author's own work, some of which has not yet appeared elsewhere.

Chapter 1 introduces the Haar-like conditions mentioned in the title, and the definitions of $H_{\mathscr{F}}$, $T_{\mathscr{F}}$, and $WT_{\mathscr{F}}$-systems, where $\mathscr{F}$ is a set of n-tuples of linear functionals. Haar,